Homework 7

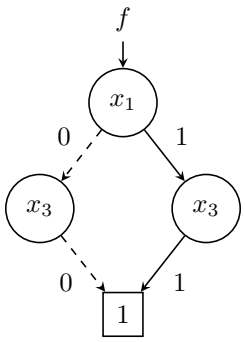
To hand in on December 5th at the beginning of the exercise session, or by email to `dghosh@lmf.cnrs.fr` by the end of the day.

Answers can be written in French or in English.

Exercise 1. Draw the BDDs for the following functions, using the order of your choice on the variables $\{x_1, x_2, x_3\}$. You may omit the 0-node. No justification is necessary.

1. $(x_1 \Leftrightarrow x_2) \vee (x_1 \Leftrightarrow x_3)$,
2. $s(x_1, x_2, x_3) = \begin{cases} 1 & \text{if } x_1 \texttt{ xor } x_2 \texttt{ xor } x_3 = 1 \\ 0 & \text{otherwise}. \end{cases}$

Exercise 2. Let $x_1, \ldots, x_n$ be Boolean variables for some $n \geq 1$. We fix the ordering $x_1 < \cdots < x_n$. Given a function f, we let $B(f)$ denote the number of nodes labelled with variables in the BDD for f. For instance, the figure below shows the BDD of $f := x_1 \Leftrightarrow x_3$, where we have $B(f) = 3$.



Depending on n, how many different functions f exist such that

1. $B(f) = 1$?
2. $B(f) = 2$?
3. $B(f) = 3$?