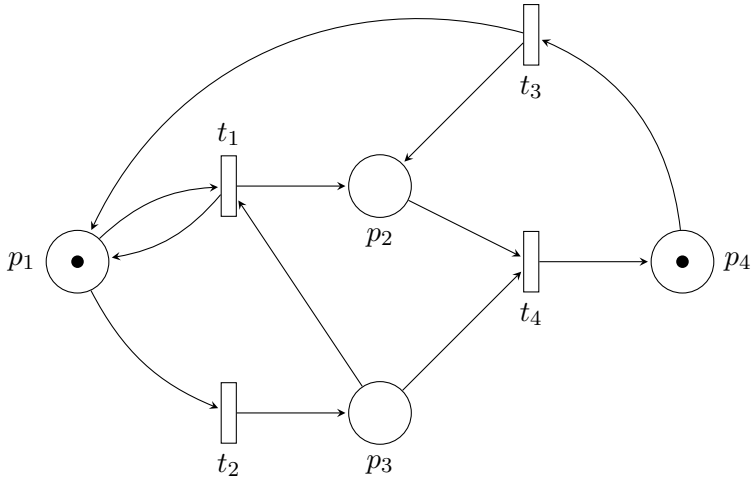
Homework 9

To hand in on December 19 at the beginning of the exercise session, or by mail by the end of the day at `dghosh@lmf.cnrs.fr`. Answers can be written in French or in English.

Exercise 1. Let $\mathcal{N}$ be the following Petri net:



1. Draw the reachability graph of $\mathcal{N}$. A marking m will be denoted by the tuple $\langle m(p_1), m(p_2), m(p_3), m(p_4)\rangle$, for instance the initial marking is $\langle 1, 0, 0, 1\rangle$.
2. Is $\mathcal{N}$ 1-safe ? 2-safe ? 3-safe ?

Exercise 2. Let $\mathcal{N} = \langle P, T, F, W, m_0\rangle$ be any Petri net and let us define the directed graph $G_{\mathcal{N}} = (P \cup T, F)$

1. Show that if $m \xrightarrow{t_1} m_1 \xrightarrow{t_2} m'$ in $\mathcal{N}$ and ${t_1}^\bullet \cap {}^\bullet t_2 = \varnothing$, then there exists a marking m_2 such that $m \xrightarrow{t_2} m_2 \xrightarrow{t_1} m'$.
2. Let $m_1 \xrightarrow{t_1} m_2 \xrightarrow{t_2} \cdots \xrightarrow{t_k} m_{k+1}$ be an execution in $\mathcal{N}$ for some $k > 1$. Assume that for all $1 < i < k$, there exists a nonempty path from t_1 to t_i in the graph $G_{\mathcal{N}}$, and that there is no nonempty path from t_1 to t_k in $G_{\mathcal{N}}$. Show that there exists an execution $m_1 \xrightarrow{t_k} m'_2 \xrightarrow{t_1} m'_3 \xrightarrow{t_2} \cdots \xrightarrow{t_{k-1}} m'_{k+1} = m_{k+1}$.