

TD 11: Petri Nets

Exercise 1 (Reachability in Petri nets). Let $N = \langle P, T, F, W, m_0 \rangle$ be a Petri net. We say that N is *acyclic* if the directed graph $\langle P \cup T, F \rangle$ does not contain any cycles. Let \mathcal{A} denote the class of Petri nets that are (i) 1-safe and (ii) acyclic.

- (a) Assuming that every transition is connected to at least one place, show that in no firing sequence of a net $N \in \mathcal{A}$ can contain the same transition twice.
- (b) Show that the reachability problem for the class \mathcal{A} is in NP.
- (c) Show that the reachability problem for the class \mathcal{A} is NP-hard, by reduction from the SAT problem. (Without loss of generality, one can assume that a SAT formula is given in conjunctive normal form.)

Exercise 2 (Traffic Lights). Consider again the traffic lights example from the lecture notes:

- (a) How can you correct this Petri net to avert unwanted behaviours (like $r \rightarrow ry \rightarrow rr$) in a 1-safe manner?
- (b) Extend your Petri net to model two traffic lights handling a street intersection. You can assume that your traffic lights are in France ($\mathbf{R} \rightarrow \mathbf{G}, \mathbf{G} \rightarrow \mathbf{Y}, \mathbf{Y} \rightarrow \mathbf{R}$).

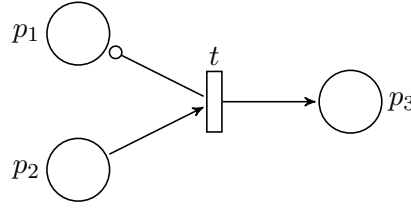
Exercise 3 (Producer/Consumer). A producer/consumer system gathers two types of processes:

producers who can make the actions *produce* (p) or *deliver* (d), and

consumers with the actions *receive* (r) and *consume* (c).

All the producers and consumers communicate through a single unordered channel.

- (a) Model a producer/consumer system with two producers and three consumers. How can you modify this system to enforce a maximal capacity of ten simultaneous items in the channel?
- (b) An *inhibitor arc* between a place p and a transition t makes t fireable only if the current marking at p is zero. In the following example, there is such an inhibitor arc between p_1 and t . A marking $(0, 2, 1)$ allows to fire t to reach $(0, 1, 2)$, but $(1, 1, 1)$ does not allow to fire t .



Using inhibitor arcs, enforce a priority for the first producer and the first consumer to access the channel: the other processes can use the channel only if it is not currently used by the first producer and the first consumer.

Exercise 4 (Model Checking Petri Nets). Let us fix a Petri net \mathcal{N} with inhibitor arcs. We consider propositional LTL, with a set of atomic propositions AP equal to P , the set of places of the Petri net. We define proposition p to hold in a marking m in \mathbb{N}^P if $m(p) > 0$.

The models of our LTL formulæ are *computations* $m_0 m_1 \dots$ in $(\mathbb{N}^P)^\omega$ such that, for all $i \in \mathbb{N}$, $m_i \rightarrow_{\mathcal{N}} m_{i+1}$ is a transition step of the Petri net \mathcal{N} .

- (a) Prove that state-based LTL model checking can be performed in polynomial space for 1-safe nets. For this, prove that one can construct an exponential-sized Büchi automaton $\mathcal{B}_{\mathcal{N}}$ from a 1-safe Petri net that recognizes all the infinite computations of \mathcal{N} starting in m_0 .
- (b) In the general case, state-based LTL model checking is undecidable for this model. Prove it for Petri nets with at least two unbounded places, by a reduction from the halting problem for 2-counter Minsky machines.

(The 2-counter Minsky machine that we consider has three operations: increment, decrement and zero test. Increment is executable unconditionally, decrement is executable only if the counter is not 0 and zero test is executable only when the counter is zero.)